Exponential Frequency Spectrum in Magnetized Plasmas

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Measurements of a magnetized plasma with a controlled electron temperature gradient show the development of a broadband spectrum of density and temperature fluctuations having an exponential frequency dependence at frequencies below the ion cyclotron frequency. The origin of the exponential frequency behavior is traced to temporal pulses of Lorentzian shape. Similar exponential frequency spectra are also found in limiter-edge plasma turbulence associated with blob transport. This finding suggests a universal feature of magnetized plasma turbulence leading to nondiffusive, cross-field transport, namely, the presence of Lorentzian shaped pulses.

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There is considerable interest in understanding the mechanisms leading to low frequency turbulence in magnetized plasmas. The term "low frequency" means that the characteristic turbulence frequency ω is smaller than the ion cyclotron frequency Ω_i. The primary reason for the interest is that turbulent fluctuations can alter the transport of mass and energy and thus degrade the performance of magnetic confinement devices that aim to achieve fusion conditions. Significant experimental and theoretical effort has been devoted to the identification of universal trends in the spectrum of fluctuations, a strong motivation being the early work by Kolmogorov [1]. Influenced by this pioneering work that predicts algebraic spectral dependencies, the published experimental plasma studies typically display the data in log-log scale. Because of the large dynamic range compressed in such displays, important phenomena underlying the turbulence are obscured. That is the situation addressed in this Letter, in which an exponential frequency dependence is highlighted. Detailed measurements of density and temperature fluctuations in a controlled temperature gradient embedded in a magnetized plasma are shown to exhibit such a spectrum. Its origin is traced to a series of individual pulses. In addition, it is found in a separate experiment that the frequency spectrum of density fluctuations at the limiter edge of a large plasma column has identical characteristics to those of the controlled temperature gradient experiment. It is noteworthy that current studies [2–4] have associated the limiter-edge phenomena with cross-field transport of plasma blobs [5–8].

Exponential frequency spectra have previously appeared in a wide range of studies but little or no attention has been devoted to their significance. Figure 1(a) of Ref. [9] exhibits exponential behavior over 4 orders of magnitude in an experiment in a helical confinement device in which proof of inverse cascade has been reported. Figure 1 of Ref. [10] displays exponential dependence in a tokamak experiment for which it is claimed that magnetic-fluctuation-induced heat transport is observed. Figure 6(b) in Ref. [11] shows coherent modes embedded in an exponential spectrum in a nonlinear dynamics experiment in a low-pressure arc plasma. Figure 7 of Ref. [12] identifies an exponential spectrum in the magnetic fluctuations at the nominal free edge of the linear device in which the present studies are performed. The work presented in this Letter supports a universal character of plasma turbulence that may account for the cited observations.

The two separate experiments reported here are performed in the Large Plasma Device (LAPD-U) [13] operated by the Basic Plasma Science Facility (BaPSF) at the University of California, Los Angeles. The experiment involving the controlled temperature gradient uses a small electron beam to create a hot electron temperature filament [14] embedded in the center of a large, cold magnetized plasma. The limiter-edge experiment uses a metallic plate inserted at the plasma edge to establish a sharp density gradient [15] in the nominal plasma column of the LAPD-U that results in a system similar to the scrape-off layer of a tokamak.

The standard LAPD-U helium plasma column is 60 cm in diameter and 17.5 m long, with density in the range of $1–3 \times 10^{12}$ cm^{-3}. The plasma source is pulsed at a 1 Hz rate and produces plasmas with a high degree of reproducibility. In the controlled temperature gradient experiment, a hot electron channel is established by injecting a small electron beam into the afterglow phase of the plasma, 0.5 ms after the main discharge voltage pulse is terminated. The axial magnetic field is uniform with a strength of 1 kG. During the main discharge the electron temperature is $T_e = 6–8$ eV, but at the time of beam injection it decays to about 1 eV due to electron heat conduction to the ends of the device, while the plasma density decays on a slow time scale of about 2 ms due to ambipolar flow. The electron beam is 3 mm in diameter and carries a current of 250 mA. It is produced by biasing a heated, single crystal of lanthanum-hexaboride to 20 V relative to the mesh anode of the device. The beam is injected at plasma center towards the cathode of the device, 15 m away. Heat conduction parallel to the magnetic field results in a hot

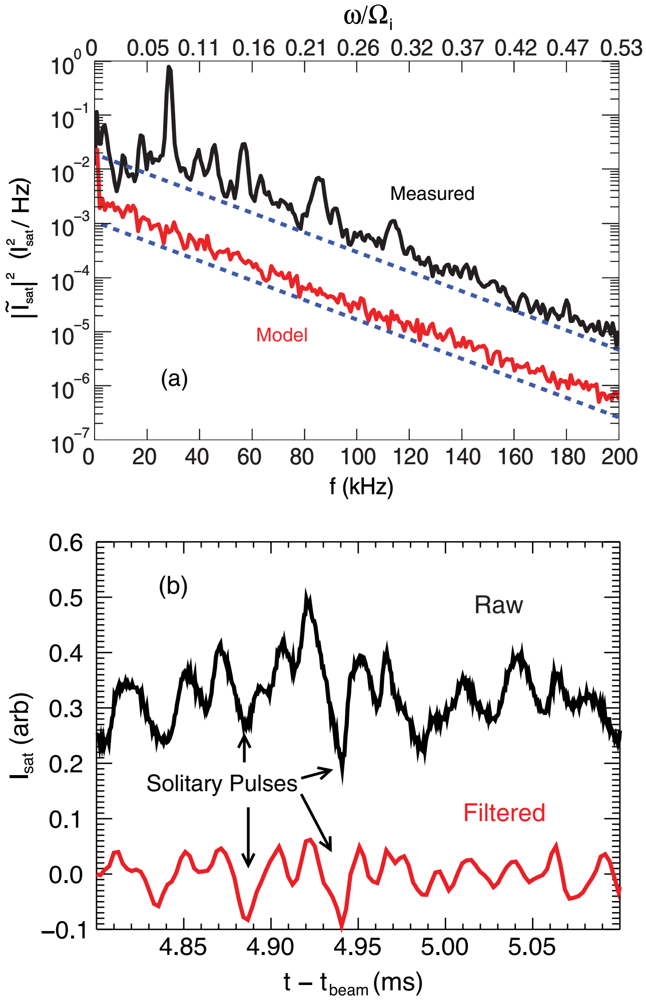

FIG. 1 (color online). (a) Power spectra of I_{sat} fluctuations (black trace) at $r = 3$ mm from the filament center displaying both an exponential character and strong coherent modes. The red or dark gray curve is a numerical simulation of an ensemble of randomly distributed Lorentzian pulses, dashed blue lines are the same exponential fit to the data. (b) Time evolution of single-shot ion saturation current (black curve) and filtered signal (red curve) showing pulses.

channel approximately eight meters in length and 5 mm in radial extent, with peak T_e of about 5 eV. The system approximates a nearly ideal situation consisting of a pure electron temperature gradient (i.e., having uniform plasma density) across the magnetic field and embedded in an infinite, colder plasma. Thus the observed turbulence and associated transport is insensitive to boundaries. This situation is a contrast to the limiter-edge environment in which the plasma is radially in close contact with a metallic object.

The temperature filament exhibits two different types of coherent oscillations: a thermal (diffusion) wave at approximately 5 kHz that is strongly localized to the filament center, and a drift-Alfvén wave [16,17] in the 25–40 kHz range whose spatial eigenfunction peaks around the maximum temperature gradient, located 3 mm from the filament center. A detailed review of the measured fluctuation profiles is provided in [16]. Initially, these modes appear as distinct, relatively narrow spectral peaks that are well separated from background noise. But later in time, a broadband spectrum develops that can mask the spectral signature of the coherent modes. Such a situation is illustrated by the top trace in Fig. 1(a) in which the spectrum of ion saturation current fluctuations $\tilde{I}_{sat}$ is shown. Ion saturation current is sensitive to both electron density and temperature, $I_{sat} \propto n_e\sqrt{T_e}$. In this semilog display the coherent modes are seen to rise above a broadband continuous spectrum (sketched by the dashed line) having an exponential frequency dependence. Exponential frequency dependence is easily recognizable as a linear dependence in a semilog format. The spectrum shown corresponds to an ensemble average over 20 discharge pulses. The top trace in Fig. 1(b) shows one of the single-shot time traces that contributes to the average spectrum of Fig. 1(a); it contains a mixture of coherent oscillations and individual pulses. The lower trace in Fig. 1(b) clearly shows the pulses extracted by filtering out the coherent modes numerically in Fourier space and reconstructing the time signal. Other filtering or search schemes yield similar results.

A normalized, temporal Lorentzian pulse centered at time t_0 and having width τ has the mathematical form

$$g(t) = \tau^2[(t - t_0)^2 + \tau^2]^{-1} \tag{1}$$

and corresponding Fourier transform

$$\tilde{g} = (\pi\tau)\exp(-\omega\tau + i\omega t_0). \tag{2}$$

The red or dark gray curve in Fig. 1(a) corresponds to a numerical simulation of an ensemble of randomly distributed Lorentzian-type pulses. The ensemble consists of 20 different trial traces, each 1 ms in duration, while each trial trace contains 10 Lorentzian pulses. These pulses are randomly centered in time with widths uniformly distributed between 2.5–4.5 μs. While the power spectrum of any single Lorentzian pulse is exponential in frequency, the exponential slope calculated from a measured data set is determined by the width distribution of the individual pulses. The model width distribution is consistent with the actual pulses found in the experiments. A fast-Fourier-transform (FFT) power spectrum is calculated for each trial trace and ensemble averaged to produce the model (red or dark gray) curve. In Fig. 1(a) the vertical offset between the experimental (black) and model (red or dark gray) exponential spectra is not fundamental but for clarity of display.

Figure 2 displays the time signature of a single pulse typical of the thousands of pulses in the data. This particular pulse occurred at a radial position 2 mm from the center. The extended wings arise because the time trace is not filtered to remove the low frequency thermal mode. The inset in Fig. 2 shows that the shape of the pulse is well represented by a Lorentzian function (red dashed curve).

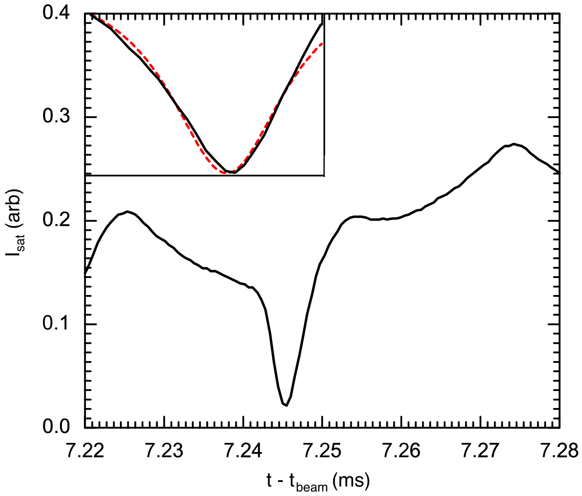

FIG. 2 (color online). Unfiltered ion saturation current highlighting a single pulse. Inset: Comparison of measured pulse (black curve) to a model Lorentzian pulse (red dashed trace).

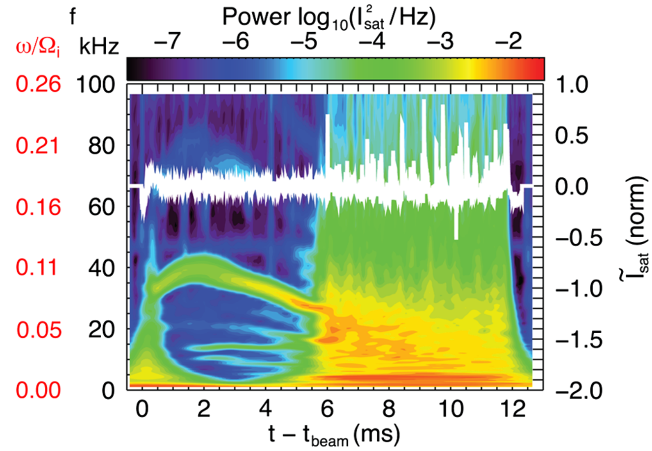

FIG. 3 (color online). Time evolution of power spectra of ion saturation current showing transition from coherent mode regime ($t - t_{beam} \leq 5.5$ ms) to broadband fluctuations displaying an exponential spectrum. Frequency is displayed on the left axis in terms of both absolute (black) and normalized (red or dark gray) units. The white trace near the top is a single-shot ion saturation current showing Lorentzian pulses when the exponential spectrum develops.

From such a fit, a numerical value for the width τ can be obtained for each pulse. This procedure results in an ensemble average value $\langle\tau\rangle = 4$ μs. Direct measurement of the exponential slope in a semilog plot of the broadband spectrum (e.g., Fig. 1(a) blue dashes) yields, $\langle\tau\rangle = 3.5$ μs, thus confirming that the underlying reason for the observed exponential spectrum is the presence of Lorentzian shaped pulses. With the Fourier transform dependence [refer to Eq. (2)] $\tilde{g} \propto \exp(-\omega\tau) = \exp(-2\pi f\tau)$, the corresponding scaling frequency of the spectrum is $f_s = (2\pi\tau)^{-1}$ with $|\tilde{g}| = (2\pi^2/f_s)\exp(-f/f_s)$. The measured value is $f_s = 53$ kHz, which is on the order of the frequency of the drift-Alfvén modes, but consistently higher. Detailed examination of the single-shot time traces indicates that the widths of the Lorentzian pulses are a fraction of a cycle of a temporally adjacent drift-Alfvén oscillation. This suggests that the pulses arise from abrupt modulations of the coherent, gradient-driven modes.

Conclusive evidence for the connection between exponential spectra, cross modulation of drift-Alfvén modes, and the appearance of intermittent Lorentzian pulses is succinctly summarized in Fig. 3. The color display corresponds to the time evolution of the ion saturation current (I_{sat}) power spectrum calculated using a continuous wavelet transform method at a radial position 9 mm from the filament center. The single white trace at the top is the fluctuating part of the ion saturation current (normalized to the nonfluctuating part) for one of the plasma discharges that make up the ensemble used to calculate the power spectrum. In the time interval $t - t_{beam} \leq 5.5$ ms the spectrum shows a temporally evolving, coherent drift-Alfvén mode at a frequency between 25–40 kHz and no broadband noise. At these earlier times classical transport due to Coulomb collisions is observed. But at about 5.5 ms, intermittent positive pulses are seen to arise. Departure from classical transport, as previously documented by Burke *et al.* in Fig. 18 of Ref. [16], is observed to occur simultaneously with the appearance of exponential power spectra. Simultaneously, with the generation of Lorentzian pulses, it is seen that a strong low frequency mode appears (close to 5 kHz) and the coherent drift-Alfvén mode disappears into a broadband spectrum. It is this broadband spectrum that exhibits the exponential frequency dependence associated with the Lorentzian pulses exemplified in Fig. 2.

Lorentzian shaped pulses are also identified in the limiter-edge experiment [15] where a steep density gradient is present. Here positive pulses (“blobs”) travel out of the core plasma while negative pulses (“holes”) travel back in, both originating in a birth region located on the steep density gradient. The measured frequency spectrum of ensembles of such pulses is displayed in a semilog presentation in Fig. 4 for a case with uniform axial magnetic field strength of 1.5 kG. The exponential character of the spectrum is evident for the three traces shown. They correspond to measurements of the ion saturation current at three different positions across the magnetic field. At the outermost position (red or dark gray trace) the pulses have positive polarity while at the innermost position (blue or gray trace) the pulses have negative polarity. At an intermediate position both positive and negative pulses are observed. A similar ordering is also observed in the electron temperature gradient experiment in spite of the radial spatial scales being considerably smaller than in the limiter-edge situation. Figure 2 shows a negative pulse in the interior of the filament ($r = 2$ mm) while the white trace at the top of Fig. 3 exhibits positive pulses in the periphery ($r = 9$ mm). Figure 5 shows one of the positive

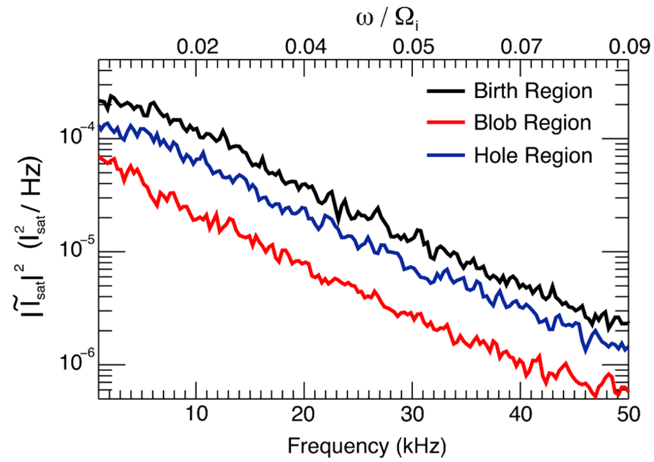

FIG. 4 (color online). Power spectra of ion saturation current in limiter-edge experiment display exponential behavior in frequency at different radial positions.

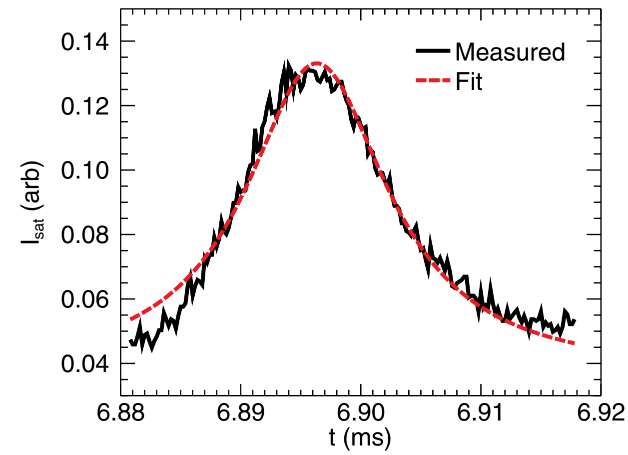

FIG. 5 (color online). Ion saturation current (solid black curve) and Lorentzian fit (dashed red curve) from the limiter-edge experiment.

pulses or "blobs" from the limiter-edge experiment (black curve) along with a Lorentzian-pulse fit (dashed red curve). It is seen that the temporal shape of individual pulses measured in the limiter-edge are also well described by a Lorentzian function, as is necessary to yield an exponential spectrum. The characteristic frequency associated with the spectra in Fig. 4 is 22.9 kHz, corresponding to a Lorentzian width of 7.0 μs, while the width resulting from curve fitting (see Fig. 5) is found to be 7.2 μs.

It is important to emphasize that the exponential nature of the spectrum in both experimental situations reported here is not a consequence of statistics, as is the case for blackbody radiation or other statistically determined spectra. Spectra of individual single shots exhibit the exponential structure.

In summary, it is shown that, in a controlled experiment, turbulence associated with temperature gradient instabilities develops an exponential frequency spectrum. The phenomenon is associated with Lorentzian shaped pulses generated by the interaction of drift-Alfvén modes ($\delta I_{sat}/I_{sat} > 20\%$) with lower frequency thermal oscillations. Density-gradient-driven turbulence at the limiter edge exhibits identical features to that of the microscopic temperature filament. Because the limiter-edge experiment does not have a well-defined initial time it is not yet possible to identify the transition from coherent to broadband behavior that is so dramatically demonstrated in Fig. 3 for the temperature filament. From the similar behavior observed in these totally different situations and the previous reports of exponential spectra by various investigators, it appears that the phenomenon is a universal feature of turbulence in magnetized plasmas. Further theoretical, modeling, and experimental studies are needed in order to conclusively establish a universal nature. In addition, the findings naturally connect properties related to blob transport to nonlinear interactions of drift-Alfvén modes.

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